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Technology for organizing mathematical problem solving using the concept of stage-by-stage development of mental actions and criteria-based assessment when training future mathematics teachers

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Abstract: Mathematical knowledge, due to the specific nature of its acquisition through problem solving, plays a special role in the development of various forms of thinking. When solving mathematical problems through the interiorisation of heuristic techniques, a cultural form of creative thinking is formed. The acquisition of analytical heuristics can be influenced by specific methods of organizing the learning process. This paper describes a tool for organizing the teaching of mathematical problem solving as a process aimed at developing guidelines – heuristics – according to the third type of orientation of P.Ya. Galperin's theory of the stage-by-stage development of mental actions. The paper presents the results of a formative experiment on the acquisition of the proposed framework (the scheme for organizing mental activity, hereinafter referred to as the OMA) by future mathematics teachers while studying the integral calculus of one variable functions. Using the Mann–Whitney *U*-test, statistically significant differences in the levels of development of the Problem Setting Analysis heuristic were obtained in the control and experimental groups. Due to the small number of groups, a qualitative analysis of the experimental results was conducted. The feasibility of using the OMA framework to implement a strategy of complete acquisition and formative assessment is demonstrated. As it is hypothesized, the systematic use of the OMA framework in teaching to solve problems implements a third-type orientation teaching method and is appropriate for training maths teachers.

Keywords: teaching to solve mathematical problems; theory of stage-by-stage development of mental actions; third type of orientation; formative assessment; full acquisition; mathematics teacher education; integral calculus of one variable functions.

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1. INTRODUCTION

The problem of mastering mathematics as a subject of study has a long history. Each new generation of students brings its own unique characteristics to the problematic area. These characteristics are related to the specific state of the educational environment, its dependence on the sociocultural environment, and the level of technological development, and therefore require immediate study.

In current environment, with the rapid development and use of electronic calculators, the ability to discover the meaning and significance of mathematical knowledge is a non-trivial challenge. For Generation Z students, the so-called digital natives, it is necessary to "construct" an understanding of mathematical text as a specially organized process. The acquisition of a course in mathematics supposes the transformation of "ready-made" mathematical knowledge recorded in textbooks, reference books, and information systems into knowledge that is "emerging" [1]. A traditional mathematical problem is a "dynamic" structure for representing mathematical knowledge in this sense. It contains a direct appeal to the problem solver in the form of a question and creates opportunities for discursive analysis of the problem situation. It is worth recognizing that in today's information richness, the educational process in its traditional form does not fully realize these opportunities – in most cases, students do not develop a model of mathematical activity[1].

The theoretical basis for the study was the activity theory of the psyche by A.N. Leontiev (1903–1979) [2], which had a significant influence on Russian pedagogy. In the context of the problem under study, of particular interest is the development of its provisions in the theory of educational activity by D.B. Elkonin (1904–1984) and V.V. Davydov (1930–1998), which laid the foundations for

[1] *Makeeva O.V., Foliadova E.V. Math teacher for generation Z: mastering the mathematical content by constructing knowledge bases. Razvitie obshchego i professionalnogo matematicheskogo obrazovaniya v sisteme natsionalnykh universitetov i pedagogicheskikh vuzov: sbornik materialov 40-go Mezhdunarodnogo nauchnogo seminara prepodavateley matematiki i informatiki universitetov i pedagogicheskikh vuzov. Bryansk, IP Khudovets R.G. Publ., 2021, pp. 416–419. EDN: KMLXNL.*

the activity-based approach in education [3; 4]. The ideas of L.S. Vygotsky (1896–1934) on the relationship between learning and mental development [5] can be considered as a unified theoretical basis for various systems of developmental learning.

The works of P.Ya Galperin (1902–1989) [6] find their origins in the research of A.N. Leontiev and his followers. Based on the orienting basis in the doctrine of activity, the scientific school of P.Ya. Galperin developed a theory of the stage-by-stage formation of mental actions. This is a theory of learning as the transition of external activity to the internal plane in the course of interiorisation, which describes the processes and conditions for the formation of meaningful actions to generate the subject's knowledge – ideas and concepts about objects and their connections [7]. These include the active orientation of the subject in the conditions of the action, where the third type of orientation (complete and generalized orienting basis of action) has fundamental importance; the presence of means of action as tools of mental activity (standards, measures, signs); understanding the process of the emergence of images of perception and thinking as the transition of external actions to the plan of operations carried out in the mind. The theory of P.Ya. Galperin is well known in Russian psychology and pedagogy; it has received wide international recognition and continues to be developed in recent research [8; 9]. Education built on its principles is more effective than traditional system, as it manages the process of developing mental actions, including the qualities of actions. Positive examples include educational programs for primary schools [6]. However, this approach is not actively used in the modern system of higher pedagogical education.

The study of psychological and pedagogical mechanisms of learning is inextricably associated with the problem of assessing the results of educational activity. Currently, the criteria-based assessment system, which is essentially formative assessment, has become quite widespread. This system is based on a six-level taxonomy of educational goals [10] developed in 1956 by the American psychologist of learning methods Benjamin Samuel Bloom (1913–1999). In the 1960s, in collaboration with the American psychologist John Bissell Carroll (1916–2003), he formulated the idea of complete acquisition. It is based on the hypothesis that complete material acquisition is accessible to every student. To achieve this, learning outcomes must be selected as an invariant, an unchangeable parameter of the educational process. The formulation of educational goals can be carried out through the description of learning outcomes expressed in the actions of students. Assessment criteria are determined by the objectives of the educational work and represent a list of the types of actions the student carries out and must master during the work. In certain situations, such a procedure is quite naturally constructed and operationalized. Currently, the formative assessment system is quite widespread abroad at all levels of education. However, for Russian schools and especially universities, this assessment strategy remains innovative [11].

The study deals with the training of future maths teachers. It focuses on the problem of organizing the process of solving educational mathematical problems that is adequate to the psychological characteristics of the process of thinking and aimed at developing a model of mathematical activity.

The goal of the study is to improve the effectiveness of developing subject-methodological competencies in future maths teachers through the development and implementation of a problem-solving teaching tool that implements P.Ya. Galperin's concept of the third type of orientation and the principles of formative assessment.

2. METHODS

2.1. The Study Tool and Theoretical Background

To organize learning activities for solving mathematical problems, in which learning corresponds to the third type of orientation and students are provided with criteria for assessing their achievements, the author developed a special structure – the Instructional Framework for Organizing Mental Activity when Solving Mathematical Problems (OMA Framework) (Table 1) [1]. The framework is intended to externalize and reflect in external speech processes that, with developed problem-solving skills, occur in a condensed form in the mind.

2.2. Objective and Hypothesis of the Experiment

To study the capabilities of the OMA Framework as a tool for organizing orientation and teaching it as an analytical heuristic, the author conducted a formative experiment. The purpose of the experiment was to investigate the potential of students to master the Problem Setting Analysis (PSA) heuristic technique (Table 1, part 1) under the guidance of a teacher through regular and meaningful use of the OMA Framework while solving problems during classroom learning. It is assumed that the systematic use of the OMA Framework in teacher-guided classroom learning has a statistically significant effect on students' mastering of the PSA heuristic technique.

2.3. Sample and Procedures

The experiment participants were first-year students majoring in 44.03.05 Pedagogical Education with two training profiles: "Mathematics. Foreign Language" (MFL, 16 students) and "Mathematics. Economics" (ME, 19 students). They were not informed that the instructor was conducting the experimental work.

2.4. Organization of Experimental Conditions

In the MFL group (the experimental group), problem solving was organized using the OMA framework. It was presented at the very beginning of learning the topic and was repeatedly updated in the process of the work. At each lesson, under the instructor's guidance, students collectively analyzed the problem conditions (Table 1, part 1). The framework was not fully utilized for all the tasks examined. In the ME group (control group), teaching problem solving was conducted in a traditional format (without using a scheme) – as commented problem solving.

Table 1. *Instructional scheme for organizing mental activity when solving mathematical problems (OMA Framework)*
Таблица 1. *Инструкционная схема организации мыслительной деятельности при решении математических задач (схема ОМД)*

A student is able to	**1. Mathematical problem setting analysis**		
	1	name	the object of study
	2	formulate	the subject of study
	3	outline	the answer to the problem (possible variants of the results of object's research)
	4	identify	the research object components
	5	characterize	the research object components in accordance with the requirements of research subject
	2. Problem solving		
	1	formulate	the key idea of problem solving (technique of object research)
	2	select	problem solving "tools" (methods of object research)
	3	comment on	the application of "tools" at each step of problem solving (process of object research)
	4	formulate	an answer to the problem (result of object research)
	5	identify	the stages of problem solving (structure of object research)
	3. Problem solving analysis		
	1	check	the correctness of each step in problem solving (correctness of the process of object research)
	2	assess	the completeness of the solution to the problem (completeness of object research)
	3	assess*	the rationality of the solution to the problem (rationality of the process of object research)
	4	formulate**	conclusions on problem solving (comprehensive results of object research)
	5	analyze***	the possibility of transferring the results of problem solution (uniqueness of the research object)

*Note. * Assessing the rationality of a solution involves comparing alternative solution options or its individual elements.*
*** Formulating conclusions on the solution involves identifying generalized solution techniques. This is performed when first encountering a certain problem type. Subsequently, the identified technique becomes a "ready-made" tool for analyzing the conditions of the problem.*
**** Analyzing the possibility of transferring solution results involves studying the "solution stability" when varying the characteristics of the problem object and is essentially an additional research task. This is performed when first encountering a certain problem type.*

*Примечание. * Оценка рациональности решения предполагает сопоставление альтернативных вариантов решения или отдельных его элементов.*
*** Формулировка выводов по решению предполагает выделение обобщенных приемов решения. Выполняется при первой встрече с некоторым типом задач. В дальнейшем выделенный прием становится «готовым» инструментом анализа условия задач.*
**** Анализ возможности переноса результатов решения предполагает исследование «устойчивости решения» при варьировании характеристик объекта задачи и по существу является дополнительной исследовательской задачей. Выполняется при первой встрече с некоторым типом задач.*

2.5. Time Parameters and Content Context

The experiment was conducted in natural conditions of a classroom setting, for 46 academic hours, during which students studied the integral calculus of one real variable functions within a Mathematical Analysis course. The course included 18 h for learning theoretical material, 20 h of problem-solving practice, and 8 h of assessment in the form of individual, independent student work.

2.6. Rational for the Choice of Subject Material

The subject material allowed clearly demonstrating the possibilities of using the scheme both in general and specifically in the first part covering the condition analysis. Each type of integrals considered during the course of integral calculus of one real variable functions requires the solver to identify a new component and/or its characteristic within its structure. Thus, when moving from indefinite integrals to definite and improper integrals, the "integration domain" component is added. When moving from definite integrals to improper integrals, the properties of the "integration domain" or "subintegral function" components change – boundedness gives way to unboundedness.

2.7. Assessment and Data Collection System

The control tests stipulated in the schedule served as the control stages of the experiment. The proportions of the maximum possible result expressed in points were used as quantitative information that reflected the level of development of the analytical Problem Setting Analysis heuristics. Step-by-step completion of a special assignment for analyzing the condition (Table 1, part 1) and the problems included in the control tests were assessed.

2.8. Limitations of the Study

Conducting the experiment within a real educational process imposed its own limitations. When summing up the results, only the results of those students who did not miss a single class were taken into account. Therefore, the conclusions of the study are based on the results of small samples: 5 people in the experimental group (MFL, 16 people) and 7 people in the control group (ME, 19 people). This significantly limited the possibilities of using quantitative analysis when processing the results.

2.9. Statistical Methods of Analysis

Preparatory Part of the Experiment

The homogeneity of the experimental and control group samples was tested using the Mann–Whitney *U*-test based on the results of an independent assessment of the level of preparation (the sum of the Unified State Exam scores for university admission). The hypothesis of the absence of statistically significant differences in the level of preparation of students in the experimental and control groups was confirmed at a significance level of p=0.05.

Main part of the experiment

The Mann–Whitney *U*-test was used to test the hypothesis that the values of the Problem Setting Analysis indicator differ in the experimental and control groups, with this indicator being higher in the experimental group.

Final part of the experiment

Based on the results of the work of the groups at three control stages, changes (shift) in the Problem Setting Analysis indicator were studied. The hypothesis being tested is that the change in the indicator is not random. The Wilcoxon rank-sum test was used to test this hypothesis. The hypothesis was not statistically significantly supported in either group.

2.10. Qualitative Analysis

Due to the small size of the groups, the study of the dynamics of the Problem Setting Analysis indicator was conducted through a qualitative analysis of the graphical presentation of the experimental results. Radar plots were used to compare individual results with the mean obtained after combining the groups.

3. RESULTS

3.1. Testing the hypothesis about differences in the PSA Indicator

Based on the results of the first control stage of the experiment (Test No. 2 on the topic "Integration of Different Classes of Functions"), the principal hypothesis of no significant differences in the PSA indicator between the experimental and control groups was confirmed at a 5 % significance level. At the second control stage of the experiment (Test No. 3 on the topic "Definite Integrals"), differences in the PSA indicator values between the experimental and control groups were confirmed at a 1 % significance level. The third control stage of the experiment (Test No. 4 on the topic "Improper Integrals") revealed statistically significant differences in the PSA indicator values between the experimental and control groups at a 5 % significance level.

3.2. Comparison of Total Average and Individual Results

At the first control stage of the experiment, the average PSA indicator value was 0.57. The individual scores of all experimental group participants except one (No. 4) exceeded the average. Two participants (No. 1 and 5) achieved the maximum possible score of 1. The score of participant (No. 4), who did not exceed the average, was very low, at 0.15 (Fig. 1).

Three participants in the control group (No. 7, 8, and 10) achieved scores above the average. Their scores differed only slightly from the average, at 0.6. The minimum score of the control group participant (No. 11), who did not exceed the average, was zero (Fig. 1).

At the second control stage of the experiment, the average value of PSA indicator was 0.5. The individual scores of all experimental group participants exceeded 0.6. None of the participants achieved the maximum possible score. The best score was 0.89. Participant No. 5, who achieved the absolute maximum score of 1 at the first control stage (Fig. 2), achieved the highest score.

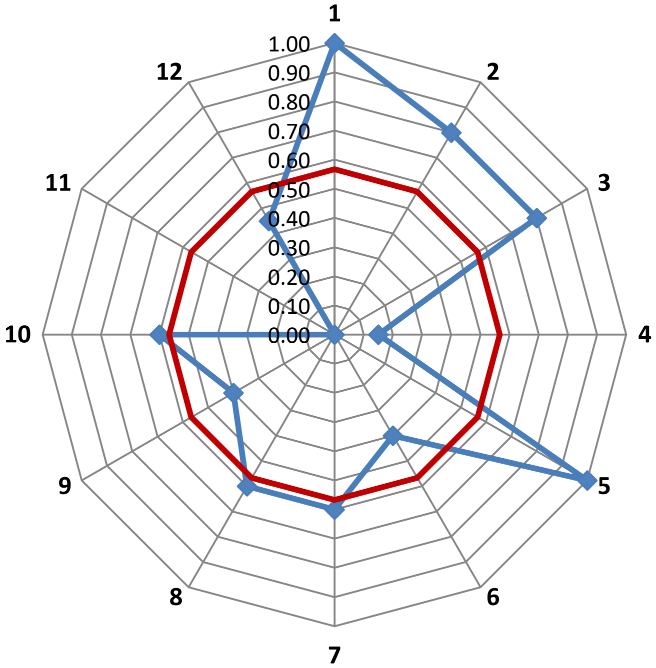

Fig. 1. *Diagram of individual results and the average value of the Problem Setting Analysis indicator at the 1st stage of the experiment.*
Numbers 1–5 indicate participants in the experimental group, and 6–12 indicate participants in the control group
Рис. 1. *Диаграмма индивидуальных результатов и среднего значения показателя «Анализ постановки задачи» на 1-м контрольном этапе эксперимента.*
Цифрами 1–5 обозначены участники экспериментальной группы, 6–12 – участники контрольной группы

Two participants in the control group (No. 7 and 10) achieved scores above the average. These scores were slightly different from the average, reaching 0.56 and 0.61, respectively. Both scores were achieved by participants who also exceeded the average score at the first control stage of the experiment. The lowest score of the control group participant (No. 12), who did not exceed the average score, was 0.17. This score did not belong to participant No. 11, who achieved the lowest score for the studied indicator at the previous control stage (Fig. 2).

At the third control stage of the experiment, the average PSA indicator score was 0.57, the same as at the first control stage. The individual scores of all participants in the experimental group, with the exception of one (No. 4), exceeded the average. Two participants (No. 1 and 2) achieved the highest possible score of 1. For participant No. 1, this was the result of both the first and third control stages of the experiment. Participant No. 2 achieved this result for the first time. Participant No. 5, who demonstrated the highest results at the first and second control stages of the experiment, had a significantly lower result at this stage – 0.81. Minimum score of participant No. 4, which did not exceed the average value, was 0.43. This participant also did not exceed the average value at the first control stage (Fig. 3).

One participant (No. 10) in the control group achieved a score above the average. His score was 0.62, slightly different from the average. This student demonstrated consistently high results throughout all three control stages of the experiment, exceeding the average. Participants No. 11 and 12 demonstrated the minimum result of 0.14. Their results were also the lowest at the first and second control stages (Fig. 3).

Finally, we present a diagram of the average individual results and the total average for all three control stages of the experiment. In the experimental group, one participant (No. 4) did not exceed the average value threshold, while in the control group, only one participant (No. 10) did (Fig. 4).

3.3. Analysis of the Dynamics of Individual Performance

An analysis of the dynamics of the PSA indicator was conducted based on the results of the three control stages of the experiment (Fig. 5). Four patterns of individual performance dynamics can be identified (Table 2).

The “up-up” pattern indicates that the individual performance at each subsequent control stage of the experiment improved compared to the previous one. This pattern was observed only in the control group and characterized 8 % of participants in the combined study group. The “down-up” pattern indicates that the decline in individual performance at the second control stage of the experiment, compared to the first, was followed by an increase in performance when moving from the second control stage to the third. This pattern is predominant for the experimental and control groups; it characterizes 50 % of participants in the experiment. The “up-down” pattern indicates that the increase in individual performance when moving from the first control stage of the experiment to the second was

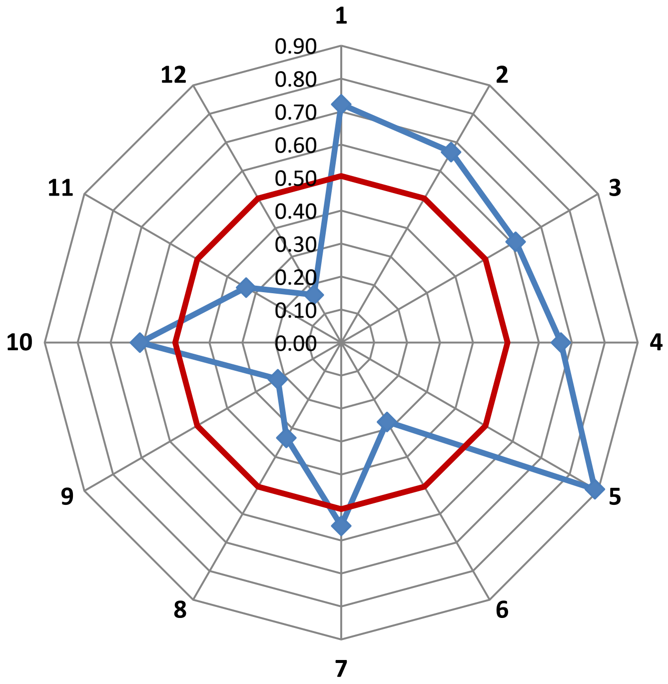

Fig. 2. *Diagram of individual results and the average value of the Problem Setting Analysis indicator at the 2nd control stage of the experiment.*
Numbers 1–5 indicate participants in the experimental group, and 6–12 indicate participants in the control group
Рис. 2. *Диаграмма индивидуальных результатов и среднего значения показателя «Анализ постановки задачи» на 2-м контрольном этапе эксперимента.*
Цифрами 1–5 обозначены участники экспериментальной группы, 6–12 – участники контрольной группы

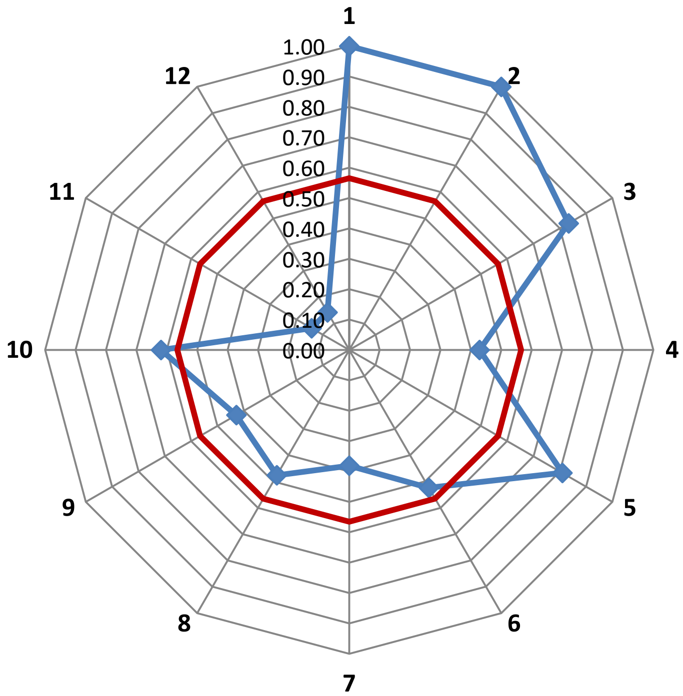

Fig. 3. *Diagram of individual results and the average value of the Problem Setting Analysis indicator at the 3rd control stage of the experiment.*
Numbers 1–5 indicate participants in the experimental group, and 6–12 indicate participants in the control group
Рис. 3. *Диаграмма индивидуальных результатов и среднего значения показателя «Анализ постановки задачи» на 3-м контрольном этапе эксперимента.*
Цифрами 1–5 обозначены участники экспериментальной группы, 6–12 – участники контрольной группы

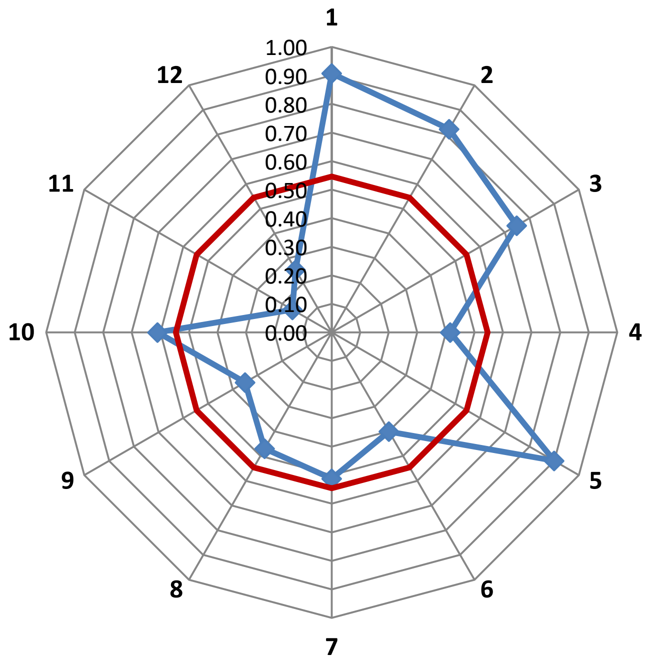

Fig. 4. *Diagram of average individual results and the overall average value of the Problem Setting Analysis indicator at three control stages of the experiment. Numbers 1–5 indicate participants in the experimental group, and 6–12 indicate participants in the control group*

Рис. 4. *Диаграмма средних индивидуальных результатов и общего среднего значения показателя «Анализ постановки задачи» на трех контрольных этапах эксперимента. Цифрами 1–5 обозначены участники экспериментальной группы, 6–12 – участники контрольной группы*

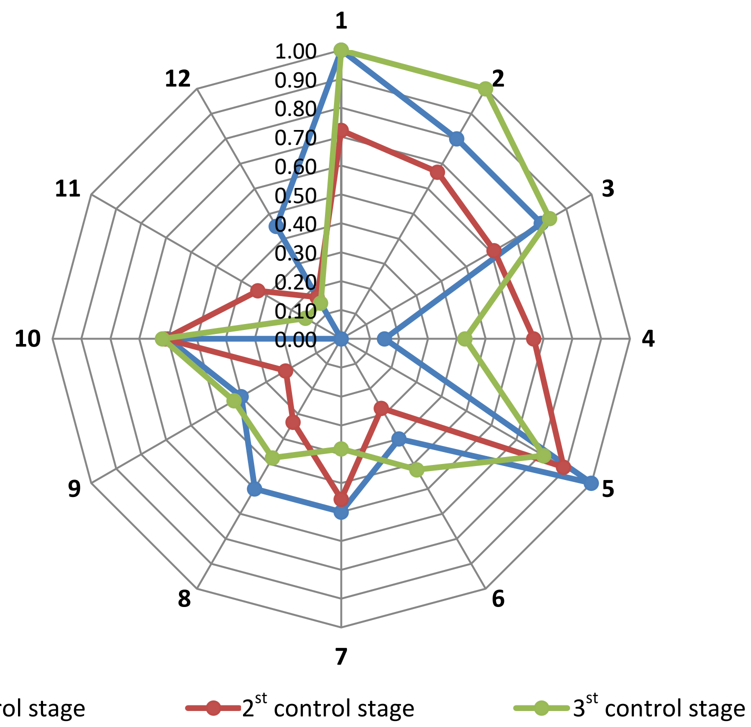

Fig. 5. *Dynamics of individual results for the Problem Setting Analysis indicator at the 1st, 2nd, and 3rd control stages of the experiment. Numbers 1–5 indicate participants in the experimental group, and 6–12 indicate participants in the control group*

Рис. 5. *Диаграмма динамики индивидуальных результатов показателя «Анализ постановки задачи» на 1-м, 2-м и 3-м контрольных этапах эксперимента. Цифрами 1–5 обозначены участники экспериментальной группы, 6–12 – участники контрольной группы*

Table 2. *Dynamics of individual results of the experiment participants*
Таблица 2. *Динамика индивидуальных результатов участников эксперимента*

Type of dynamics	Group					
	Experimental		Control		Combined	
	people	%	people	%	people	%
Up – up	0	0	1	14	1	8
Down – up	3	60	3	43	6	50
Up – down	1	20	1	14	2	17
Down – down	1	20	2	29	3	25
Control totals	5	100	7	100	12	100

followed by a decrease in performance when moving from the second control stage to the third. This trend was observed in each group and characterized 17 % of the combined group of participants. The most negative "down-down" trend indicates that each subsequent result was worse than the previous one. It was observed in both groups and characterized 25 % of the experiment participants.

3.4. Analysis of Extreme Individual Results

The author studied the dynamics of the individual extreme (best and worst) values of the PSA indicator for participants in the experimental and control groups (Fig. 5).

In the experimental group, the extreme best results did not have a clear trend. Thus, participants No. 1 and 2 exhibited fluctuations in the "down-up" dynamics, with No. 2 eventually reaching an absolute maximum and No. 1 returning to it; participant No. 5 demonstrated a negative "down-down" dynamics. The speed of movement of all participants also varied: participant No. 1 moved approximately evenly – almost three intervals in both directions; participant No. 2 moved one interval in the downward direction and three intervals in the upward direction; participant No. 5 moved one interval in the downward direction at each control stage. An interval is defined as one of the 0.1-length intervals into which the range of possible values of the PSA indicator is divided: 0–0.1; 0.1–0.2; …; 0.9–1. Participant No. 4, with the worst extreme result, also demonstrated fluctuations in dynamics: up (five intervals) and down (two intervals).

In the control group, the best extreme result of participant No. 10 was stable (it did not exceed one interval throughout all three control stages of the experiment). The worst extreme results did not show a clear trend. Thus, participant No. 11 demonstrated fluctuations in dynamics: up from absolute zero (by three intervals) – down (by two intervals); participant No. 12 – down (by three intervals) – down (remaining in the same interval).

4. DISCUSSION

Let us illustrate an example of using the OMA scheme using a problem on the topic "Indefinite Integral".

Problem 1. Find the integral $\int x \cdot \cos\left(1 - x^2\right) dx$. To correlate the process of finding a solution with the elements of the scheme, information is presented in the form of Tables 3 and 4.

The proposed structure is a flexible guideline, not a rigid algorithm. This means that nonlinear progression through the scheme is possible during the solution process, returning to previously completed points to clarify their content. Following the scheme is aimed at getting a complete, detailed, and meaningful solution. When introducing a new type of problem, the third part of the scheme is particularly important, as it helps the student develop a new solution tool where a mathematical object (class of objects) is connected with an effective method for investigating it.

The scheme was developed as a tool for working with routine problems, but the author sees its potential for mastering heuristic techniques in solving creative problems, such as those posed in Olympiad Mathematics. Using the scheme is intended to help students initiate the problem-solving process and overcome the frequently encountered "I don't know where to start" problem.

When using the OMA framework, several tasks are simultaneously and purposefully considered: an individual mathematical problem presented in the statement of the problem; the task of finding a solution to this problem; incorporating this problem into a more general class of problems; and independently posing new questions in the described problem situation. These aspects are fundamental for developing the subject and professional competences of future maths teachers.

Drawing on the works of P.Ya. Galperin and his scholars, one can list the distinctive characteristics of the learning outcome responsible for the development of the third type of orientation in students. These include: a) high rationality of action; b) high stability of action; c) broad transfer, including beyond the boundaries of the intended subject area; d) "internal", cognitive motivation; e) optimazation of the learning process; f) a true "sense of the subject", which determines

Table 3. *Solution of problem No. 1 according to the Organizing Mental Activity Framework*
Таблица 3. *Решение задачи 1 согласно схеме организации мыслительной деятельности*

	No.	**Description of the student's thinking activity in general**		**The result of performing the thinking activity**
A student is able to	**1. Mathematical problem setting analysis**			
	1	name	the object of study	indefinite integral
	2	formulate	the subject of study	finding the integral value
	3	outline	the answer to the problem (possible variants of the results of object's research)	expression of the form – $F(x)+C$; the set of all antiderivatives of the subintegral function
	4	identify	the research object components	integration variable
				subintegral function
	5	characterize	the research object components in accordance with the requirements of research subject	$x \in D(f)=R$, does not match the $\varphi = 1-x^2$ argument of $g(\varphi)=\cos\varphi$ function
				$f(x)=x\cdot\cos(1-x^2)$ – the product of two factors, where one factor is the derivative of the argument of the other factor (with an accuracy to a multiplicative constant): $(1-x^2)' = -2x$
	2. Problem solving			
	1	formulate	the key idea of problem solving (technique of object research)	representation of the integration element in the form $g(\varphi(x))\cdot\varphi'(x)dx = g(\varphi)d\varphi$
	2	select	problem solving "tools" (methods of object research)	bringing a variable under the differential sign
	3	comment on	the application of "tools" at each step of problem solving (process of object research)	see table 4
	4	formulate	an answer to the problem (result of object research)	$-\frac{1}{2}\sin(1-x^2)+C$
	5	identify	the stages of problem solving (structure of object research)	1) integration element transformation; 2) direct integration; 3) formulation and writing of the answer
	3. Problem solving analysis			
	1	check	the correctness of each step in problem solving (correctness of the process of object research)	no errors were found during the check
	2	assess	the completeness of the solution to the problem (completeness of object research)	the solution is excessively detailed: as experience is gained, items 2–4 and 5–7 can be combined* into a single item (table 4)

Table 3 continued
Продолжение таблицы 3

	No.	Description of the student's thinking activity in general		The result of performing the thinking activity
A student is able to		**3. Problem solving analysis**		
	3	assess	the rationality of the solution to the problem (rationality of the process of object research)	the solution is rational; the alternative is finding a solution using the variable substitution method
	4	formulate	conclusions on problem solving (comprehensive results of object research)	the method of "bringing a variable under the differential" sign is a generalization of the property of the indefinite integral $\int f(x)dx = F(x)+C \;\Rightarrow$ $\int f(ax+b)dx = \frac{1}{a}F(ax+b)+C$ for $\varphi(x) \neq ax+b$
	5	analyze	the possibility of transferring the results of problem solution (uniqueness of the research object)	this method can be used in cases when the integration element can be represented as $k \cdot g(\varphi(x)) \cdot \varphi'(x)dx$, i.e., the integration element is a product where one factor is a complex function $g(\varphi(x))$, and the other factor $k \cdot \varphi'(x)$ is, with an accuracy to a multiplicative constant, the derivative of the argument of the first factor

*Note. * This is a stage-by-stage transition of mental actions to the internal plane.*
The teacher controls the following stages: 1) motivation; 2) formation of an orienting basis for future action; 3) materialized actions; 4) external speech actions.

*Примечание. * Речь идет о поэтапном переходе мыслительных действий во внутренний план.*
Под управлением учителя находятся этапы: 1) мотивация; 2) формирование ориентировочной основы будущего действия; 3) материализованные действия; 4) внешнеречевые действия.

***Table 4.** Commenting on the step-by-step solution to problem No. 1*
***Таблица 4.** Комментирование пошагового решения задачи 1*

No.	Action	Commenting
1	$\int x \cdot \cos\left(1-x^2\right)dx =$	Let's rewrite the problem statement
2	$-\frac{1}{2} \cdot \int (-2x) \cdot \cos\left(1-x^2\right)dx =$	Multiply and divide the integration element by (-2) and place the constant factor $\left(-\frac{1}{2}\right)$ outside the integral sign
3	$-\frac{1}{2} \cdot \int \cos\left(1-x^2\right) \cdot \left(1-x^2\right)' dx =$	Represent the subintegral function as a product of two factors, one of which is equal to the derivative of the argument of the other factor
4	$-\frac{1}{2} \cdot \int \cos\left(1-x^2\right) \cdot d\left(1-x^2\right) =$	Bring the variable $1-x^2$ under the differential sign

Table 4 continued
Продолжение таблицы 4

No.	Action	Commenting
5	$-\frac{1}{2}\cdot\int\cos\varphi\cdot d\varphi\Big\vert_{\varphi=\varphi(x)=1-x^2}=$	Represent the integration element as $g(\varphi)\cdot\ d\varphi$, where $\varphi=\varphi(x)=1-x^2$
6	$-\frac{1}{2}\cdot\sin\varphi\Big\vert_{\varphi=\varphi(x)=1-x^2}+C=$	Find the integral using the formula from the table of basic integrals (the argument of the subintegral function coincides with the integration variable)
7	$-\frac{1}{2}\cdot\sin\left(1-x^2\right)+C$	Return to the original integration variable x using the formula $\varphi=1-x^2$

the specific approaches to solving its problems using its inherent means [12]. The use of the OMA framework, as a highly generalized and universal tool for organizing learning, is intended to create the conditions for the emergence of the aforementioned phenomena of the third type of orientation in educational activities related to solving mathematical problems [1; 13].

Organizing mathematical problem-solving activities according to the OMA framework includes:

– orientation of the problem solver in the conditions of the activity (1. Analysis of the mathematical problem setting);

– updating, selection, and application of the activity’s tools (2. Problem solving);

– comprehension of the activity being performed and its prospects (3. Analysis of the problem solution).

In other words, it allows implementing a learning model with the third type of orientation, whereby, through analysis of the essential properties and relationships of objects, students master the generalized orientation basis both for a single action and for the activity as a whole. Following the periodization of concepts of orientation within the theory of stage-by-stage action formation and discussing the characteristic distinction between orientation levels of the modern era – strategic, tactical, and operational-technical – then the OMA framework can be positioned as a working tool for organizing strategic-level orientation [8].

Representing a problem situation as a task with the inclusion of interrelated parameters is a culturally acquired psychological tool for thinking. The development of the processes of solving intellectual problems in ontogenesis is associated with the appropriation of the cultural and historical experience of productive thinking as a set of diverse heuristics [14]. The systematic use of the OMA framework in problem solving can be interpreted as a teaching technology when the acquisition of analytical heuristics becomes a manageable process.

In the practice of Russian comprehensive schools, elements of criterion-based learning have become widespread. For example, for assessing student achievement in mathematics, the “knowledge and understanding”, “research”, “communication”, and “reflection” criteria are traditionally distinguished [15]. The author’s description of the criteria does not fully coincide with the traditional one and is presented in Table 5 [13]. Due to the ambiguity of the “research” criterion, we will distinguish between the research type of the task itself and research as a method for solving problems that are not research-based in the generally accepted sense and are aimed at mastering mathematical constructs. The instructional OMA framework can serve as a tool for assessing and self-assessing learning outcomes, as it allows analyzing the level of development of specific mental processes.

Let us correlate the learning process steps proposed in the OMA framework (Table 1) with one of the criteria of student achievement assessment: *A*, *B*, *C*, *D* (Table 5) and one of the forms of information-to-knowledge transformation: D – distribution, O – organization, C – classification, V – information verification [13]. This allows comparing the steps of finding solution by complexity and move on to a quantitative analysis of student performance. Moreover, the instructional framework elements can be interpreted as descriptors describing the maximum level of the criteria-based scale for assessing student achievement (Table 6).

A qualitative analysis of the experimental results allows revealing individual differences in the PSA indicator. This demonstrates the influence of the individual characteristics of the experiment participants on the progress made. Possible causes of these differences (motivation for learning, self-regulation characteristics, individual cognitive style, etc.) could be the subject of independent research.

5. CONCLUSIONS

At a 5 % significance level, the hypothesis that special organizational techniques influence the development of students’ orientation in learning activities related to solving mathematical problems was confirmed.

The study demonstrated the feasibility of using the developed tool for organizing the teaching of mathematical problem solving using the concept of the stage-by-stage development of mental actions based on the third type of orientation for organizing criteria-based assessment of student achievement.

The developed tool for organizing learning can be recommended for use in the educational process of training

Table 5. General criteria for assessing students' achievements in the "Mathematics" subject block
Таблица 5. Общие критерии оценивания достижений обучающихся в предметном блоке «Математика»

Criterion designation	Criterion name	Criterion description
A	Knowledge and understanding	The student is able to use the language of mathematics, its laws, regularities, terms, and concepts; apply information to solve problems in familiar and unusual situations
B	Research	The student is able to select and apply appropriate mathematical knowledge, skills, and abilities to solve problems using mathematical modeling techniques
C	Communication	The student is able to concisely and mathematically correctly convey information on planning, conducting, and describing research results in oral and written communications
D	Reflection	The student is able to analyze and summarize a research problem; justify the obtained results and verify their accuracy; point out interdisciplinary connections, if any

Table 6. Correlating the solution steps in the Organizing Mental Activity framework with the general criteria for assessing students' achievements and forms of information transformation
Таблица 6. Соотнесение шагов решения в схеме организации мыслительной деятельности с общими критериями оценивания достижений обучающихся и формами преобразования информации

Learning Outcome in Terms of Activity				Assessment criterion (Table 5)	Information transformation
A student is able to	**1. Mathematical problem setting analysis**			ABD	DOC
	1	name	the object of study	B	C
	2	formulate	the subject of study	B	C
	3	outline	the answer to the problem (possible variants of the results of object's research)	D	O
	4	identify	the research object components	A	D
	5	characterize	the research object components in accordance with the requirements of research subject	A	D
	2. Problem solving			ABCD	DO
	1	formulate	the key idea of problem solving (technique of object research)	B	O
	2	select	problem solving "tools" (methods of object research)	A	O
	3	comment on	the application of "tools" at each step of problem solving (process of object research)	AC	OD
	4	formulate	an answer to the problem (result of object research)	C	O
	5	identify	the stages of problem solving (structure of object research)	D	O

Table 6 continued
Продолжение таблицы 6

Learning Outcome in Terms of Activity				Assessment criterion (Table 5)	Information transformation
A student is able to	**3. Problem solving analysis**			ABCD	DOCV
	1	check	the correctness of each step in problem solving (correctness of the process of object research)	D	V
	2	assess	the completeness of the solution to the problem (completeness of object research)	D	V
	3	assess	the rationality of the solution to the problem (rationality of the process of object research)	D	V
	4	formulate	conclusions on problem solving (comprehensive results of object research)	ABCD	DOC
	5	analyze	the possibility of transferring the results of problem solution (uniqueness of the research object)	ABCD	DOC

Note. The letters indicate the leading forms of information transformation into knowledge for a given stage of work with the task: D is distribution, O is organization, C is classification, V is information verification.

Примечание. Буквами обозначены ведущие для данного этапа работы с задачей формы преобразования информации в знание: D – разнесение; O – организация; C – классификация; V – проверка информации.

future mathematics teachers when acquiring the material of the "Integral Calculus of One Real Variable Functions" topic within Mathematical Analysis discipline.

The study confirmed the author's main idea: the developed structure for organizing the process of thinkingis an effective tool for teaching mathematical problem solving. The Organizing Mental Activity Framework can be used as a technological tool for organizing the process of problem solving teaching when training mathematics teachers.

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Технология организации решения математических задач с использованием концепции поэтапного формирования умственных действий и критериального оценивания при обучении будущих учителей математики

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Аннотация: Математическое знание в силу специфики его освоения через решение задач играет особую роль в процессе развития различных форм мышления. В ходе решения математических задач за счет интериоризации эвристических приемов происходит формирование культурной формы творческого мышления. На усвоение аналитических эвристик могут оказывать влияние специальные приемы организации процесса обучения. В работе описан инструмент организации обучения решению математических задач как процесса, направленного на формирование ориентиров – эвристик согласно третьему типу ориентировки теории поэтапного формирования умственных действий П.Я. Гальперина. Приведены результаты формирующего эксперимента по освоению предложенной конструкции (схемы организации мыслительной деятельности, далее – ОМД) будущими учителями математики в процессе изучения интегрального исчисления функций одной переменной. С помощью *U*-критерия Манна – Уитни получены статистически значимые различия в уровнях сформированности эвристики «Анализ постановки задачи» в контрольной и экспериментальной группах. В связи с малочисленностью групп проведен качественный анализ результатов эксперимента. Показана возможность использования схемы ОМД для реализации стратегии полного усвоения и формирующего оценивания. Как предполагается, систематическое использование схемы ОМД в процессе обучения решению задач реализует технологию обучения с третьим типом ориентировки и целесообразно при подготовке учителей математики.

Ключевые слова: обучение решению математических задач; теория поэтапного формирования умственных действий; третий тип ориентировки; формирующее оценивание; полное усвоение; подготовка учителя математики; интегральное исчисление функций одной переменной.

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